# Decoding Da Vinci's Sketch to the Ottomans: Galata Bridge

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**Leonardo Da Vinci's list of accomplishments put him among the world's greatest artists and inventors. However, over 500 years ago, one controversial sketch wasn't as much appreciated and has been the topic of many contemporary investigations. Today, more than five centuries later, we are reexamining Da Vinci's ambitious proposal but not only to look into the structural feasibility of whether or not his design would have been safely constructed, but to delve into the inner workings of Da Vinci's mind to see if the polymath had prior knowledge of creating stable and efficient forms which has only recently being developed using a computational framework based on the principle of geometrical equilibrium in 3D. Was his sketch just free-handed, something he had done in seconds? Or the renaissance painter and inventor had an intuition that was more than five centuries ahead of its time? Although most historians believe he had no mathematical or geometrical calculation in his design, our study proves otherwise! Through rigorous analysis of Da Vinci's design, we have found that the polymath had intuitively drawn his sketch according to the principles of geometric design that was developed in 2D almost 400 years after his time and just recently in a 3-Dimensional manner with the help of computational frameworks. This research further continues to explore the potential of Da Vinci's design with the use of modern materials and methods of construction to see how the design would have been built in our modern time.**

## 1. INTRODUCTION

Leonardo Da Vinci's list of accomplishments put him among the world's greatest artists and inventors. However, over 500 years ago, one controversial sketch wasn't as much appreciated and has been the topic of many contemporary investigations. Da Vinci's design for what would have been the longest bridge in the world was panned in the 1500s. The bridge would have connected Istanbul to the neighboring city of Galata as commissioned by an Ottoman Sultan. In Da Vinci's ambitious design, builders would have for the first time erected a bridge using a double-curvature arch. Back then, conventional bridge designs were made in the form of semicircular arches, however, Da Vinci's design was nothing like his fellows. The Ottoman Emperor rejected Da Vinci's design, and called it a 'risky endeavor,' as the polymath spelled out his pitch for the contract in a letter sent to the Ottomans, describing the bridge as being as tall as a building so that it would have allowed ships to cross underneath it without obstruction. Da Vinci was so convinced by his project that he had even offered to build it himself. The basis of the construction - three arches supporting a walkway—was first accepted as an engineering rationale 300 years after Da Vinci drew his sketch, confirming his reputation as a man ahead of his time.

Today, more than five centuries later, we are reexamining Da Vinci's ambitious proposal but not only to look into the structural feasibility of whether or not his design would have been safely constructed, as the latter has been recently proven in an MIT research using small scale experimentation,[1] but to delve into the inner workings of Da Vinci's mind to see if the polymath had prior knowledge of creating stable and efficient forms which has only recently being developed using a computational framework based on the principle of geometrical equilibrium in 3D. Was his sketch just freehanded, something he had done in seconds? Or the renaissance painter and inventor had an intuition that was more than five centuries ahead of its time? Although most historians believe he had no mathematical or geometrical calculation in his design, our study proves otherwise! Through rigorous analysis of Da Vinci's design, we have found that the polymath had intuitively drawn his sketch according to the principles of geometric design that was developed in 2D almost 400 years after his time and just recently in a 3-dimensional manner with the help of computational frameworks.

Inspired by nature, Robert Hooke in 1676, was the first mathematician who used a hanging chain to obtain the ideal form in tension. He fixed a chain on both ends, which due to gravity formed an upside-down arch depicting the force transferred in tension. He then inverted the same structure, creating a perfect form in compression. This is a funicular solution that minimizes the amount of bending where the result becomes the "line of thrust" in structure. Hooke states, "As hangs the flexible line, so but inverted will stand the rigid arch."His discovery of the physical definition of forms by nature in tension and compression was further explained through math. Seventy two years

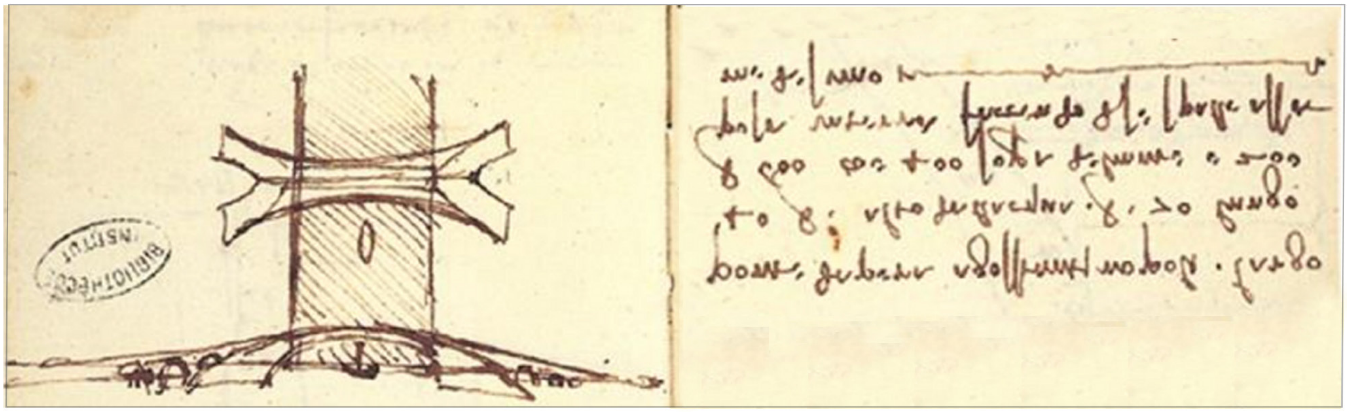


Figure 1. The Galata bridge sketch appears in one of Leonardo's notebooks in a royal library in France. (Institut de France)

later, Giovanni Poleni (in 1748) used this idea for the first time to explain the cause of the cracks in one of the large ribs of St. Peter's in Rome (built in 1585). The method in generating 3D structures was then embraced by one of the most famous architects, Antonio Gaudi, who began analyzing the different iterations and their consequences in models of various scales. Following the same concept a new method was invented to find the relationship between form and force with the help of the geometric representation of force within polygons. This was first discovered by french mathematician, Pierre Varignon,through his observation of cables and their applied forces. [2]

Following the same method, to design structures more efficiently, German structural engineer, Carl Culmann, developed a geometrical design method to generate the form of structures called graphic statics.[3] Architects borrow this idea to manifest designs with magnificent structures. One of the most famous structures designed using this technique is the Eiffel Tower in Paris. The theory of three-dimensional graphic statics was proposed by William John Macquorn Rankine in the 19th Century.[4] The idea was way ahead of its time that the method was not able to be delivered by the tools they had. Rankin proposed the idea of the reciprocal relationship between the form and force in three dimensions and it was not manifested by people at the time due to its complexity. The method has been recently reestablished by Akbarzadeh using a computational framework.[5] The first analysis and construction of 3D structures using this method was also published by the the author.[6] We also have been working on designing a framework for bridge design using the dimensions given in Da Vinci's drawing. The results of our method was accurately matched with Da Vinci's concept, which raised the question of how did he actually design it? Was he aware of mathematical and geometrical aspects of his design? If not, how did he come up with such a complex geometry? We may not know the answer, but altogether our study once again proved how intricate and complex Da Vinci's mind was.

## 2. DESIGN

The design process was inspired by Da Vinci's sketch [Figure 1]. Through following his orthodox method of designing, using a floor plan and a section, we proceeded with our formfinding process in a similar fashion. In the first stage of form-finding, a single funicular layer was produced. Consequently, as it appears in the sketch, the bridge has a double curvature, hence a second funicular layer was generated.

## 2.1 FORM-FINDING

Given the limited resources of the time period of Da Vinci's proposal to the Sultan, the design was conceived to be built with either brick or stone, which work only in compression. Hence, the primary goal of the form-finding process was to replicate Da Vinci's geometrical configuration, as accurately as possible. Graphic statics in 2D uses force diagram, with the help of at least one given load to determine the force in other members. In this method, a perpendicular / parallel lines can be drawn to the existing members which result in a form of closed polygons for each node. Through this process, some similarities between the total force of the polygon and the force within each member are obvious. The method avoids trial and error while introducing accuracy and efficiency. In two dimensions, the force or the form diagram can be generated interchangeably using basic geometry. In three dimensions however, it is more complicated. Each node in the form will be represented by a closed polyhedron and the surface area of each face is associated with the amount of the force in each member. Therefore, the reciprocal relationship between the form and force diagram allows for adjustment of the amount of design loads as well as the geometry of the form. Due to the complex nature of the problem in 3D computer aided method will be required to design the structure. PolyFrame for Rhinoceros was used in order to generate a form that works only in compression.[789] Through experimentation with 2-Dimensional patterns then extruding them to a single node, an equilibrated structure (force diagram) is created by a closed polyhedron or a polyhedral cell with planar faces. Each face of the force diagram is perpendicular to an edge in the form diagram, and the

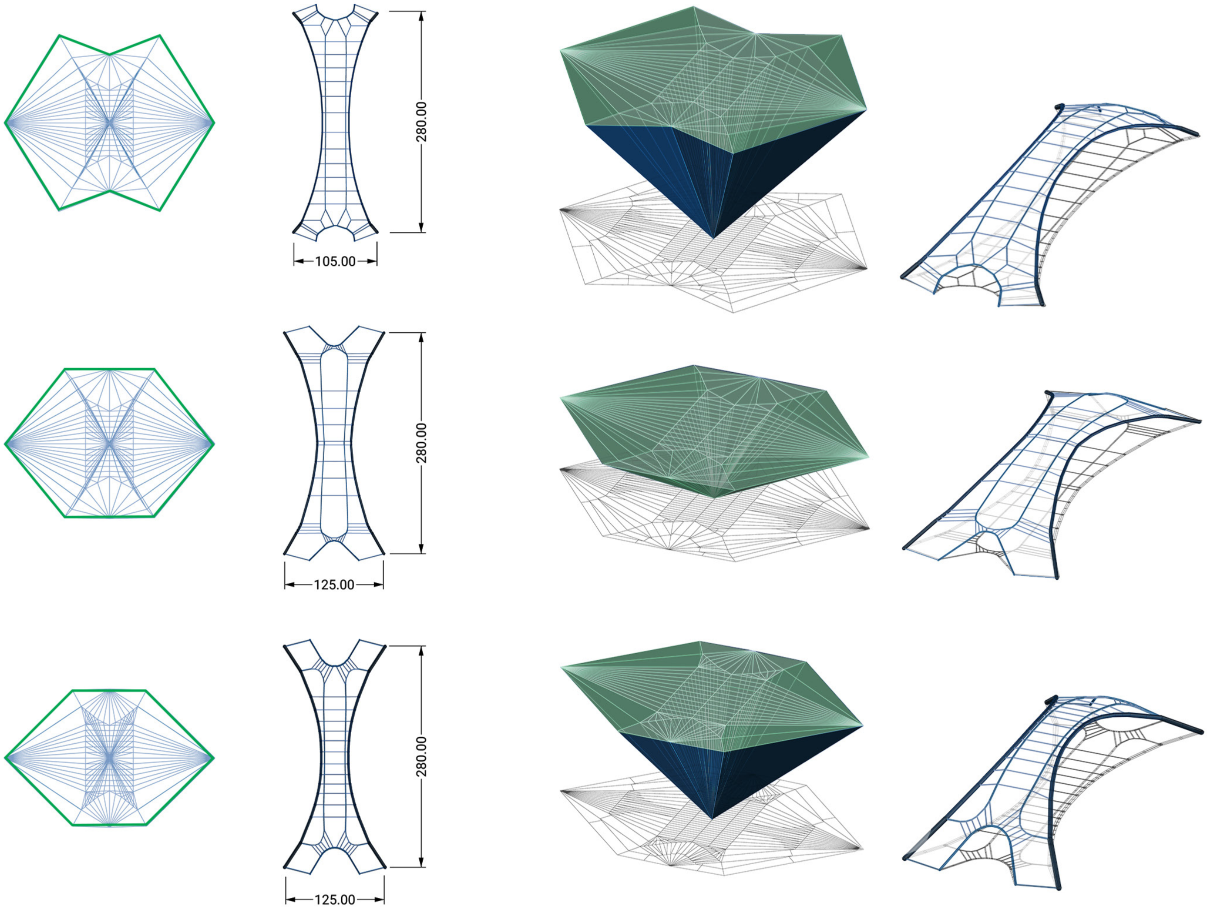


Figure 2. The form-finding of a single-layer bridge form, resulting in reaching an identical representation of Da Vinci's sketch.

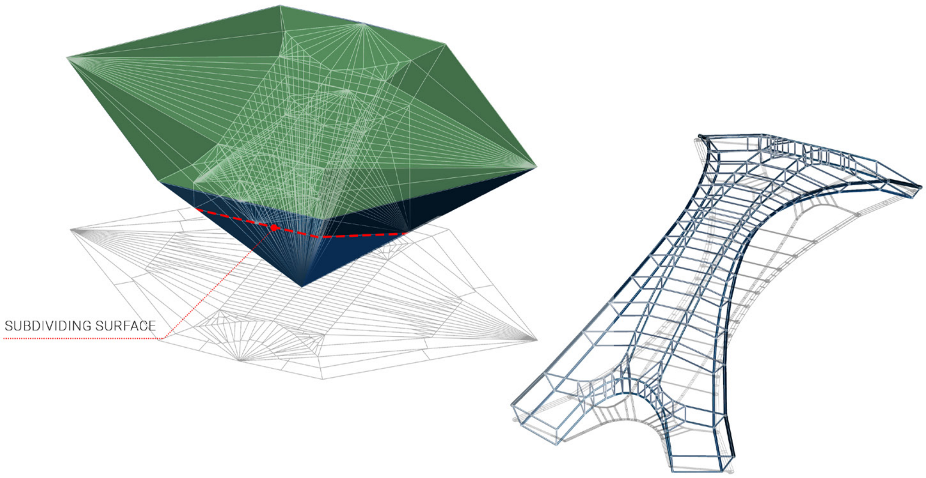
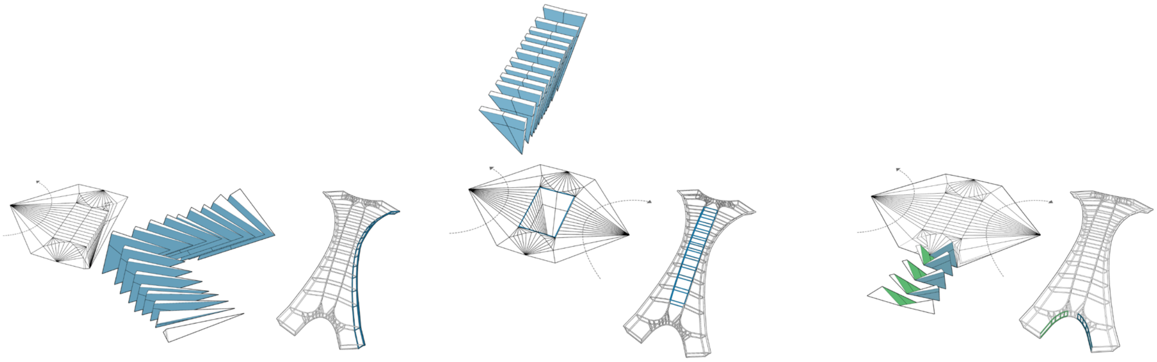


Figure 3. Transform the single-layer form to double-layer by subdividing the force polyhedrons.



SIDE CURVATURE DEMONSTRATION

CORE CURVATURE DEMONSTRATION

PARABOLIC CURVATURE DEMONSTRATION

Figure 4. Steps illustrating the configuration of the members, creating the side, core, and parabolic curvature of the geometry.

magnitude of the force in the corresponding edge is equal to the area of the face in the force polyhedron.

### 2.1.1 FORM-FINDING OF A SINGLE-LAYER FUNICULAR BRIDGE

As demonstrated below [Figure 2], a series of exploration made in order to find the most efficient, accurate geometry of a single-layer funicular bridge. Surprisingly, the hand-sketch made by the polymath is almost identical to the structure exemplary created by the computational software. Generally, there are two common methods to manipulate force diagrams, polyhedron aggregation and polyhedron splitting, both result in transforming the topology and design features in a derived form diagram. The bridge force diagram is constructed by extrusion of a 2-dimensional pattern of polygonal faces into a point [Figure 2], where the 2D polygonal faces define the curvature of the bridge and the load distribution, and the extrude length determines the height of the bridge.

### 2.1.2 DOUBLE-LAYER FUNICULAR BRIDGE BY SUBDIVIDING FORCE POLYHEDRONS

After obtaining the desired geometry through experimenting with a single layer curvature, a double curvature can be realized through a subdividing methodology to applied to the force diagram [Figure 3]. In order to convert the geometry from a single layer to a double layer, all the force polyhedrons were subdivided into two halves by a surface near the centroid of the force diagram, as shown in [Figure 3]. Consequently, all the faces are planarized to enforce the flatness of the polygonal faces. All polygonal faces are converted to polyhedral cells. The curvature of the splitting surface is a decisive parameter that affects the final geometry, because it is related to the inclination angle of the contact surfaces of adjacent cells in the double-layer topology [Figure 4].

## 2.2 MATERIALIZATION

The bridge was segmented in multiple parts to simplify the final construction and assembly. Due to the complex geometry of found pieces we have decided to build the bridge using concrete. This way, the mold can be 3D printed and simply be poured with concrete. Different concrete mixtures were developed to ensure the strength and durability of the final bridge. Fine aggregate, superplasticizer, polypropylene fiber, cement, and fume silica was used for the mixture. Constructing the bridge with thin individual units was applied using a modular method. This helps simplifying the construction of the bridge to be by assembling 50 units in total. Furthermore, the modular construction approach has the advantage of improved quality control of the prefabricated units, and accelerated construction time.

## 3. FABRICATION

In the first step all parts of the bridge were 3D printed with PLA as a hollow units. Next, they were cast with concrete and demolded after a day. All the units were polished before assembly. Instead of using a solid false work, individual struts were designed for assembly of the bridge. Supports are the only elements that were not made of concrete, instead they were printed as a solid plastic parts.

### 3.1 MOLD PREPARATION, FALSEWORK, AND ASSEMBLY

In order to convert the geometry from funicular to shellular, a set of 3D printing molds needed to be created. The force diagram produced by PolyFrame was used to achieve the latter. As showing in [Figure 5], the lines from the force diagram were used to create the surface boundaries of the mold pieces with an open top. Later each mold piece was 3D-printed separately and was given thickness of 2mm [Figure 5]. In order to assemble the bridge thirteen short columns with a top part that follows the geometry of the bridge's deck was designed. [Figure 5] shows the supports and falsework before the assembly of the bridge. Each column connects all the three individual units on top. For the

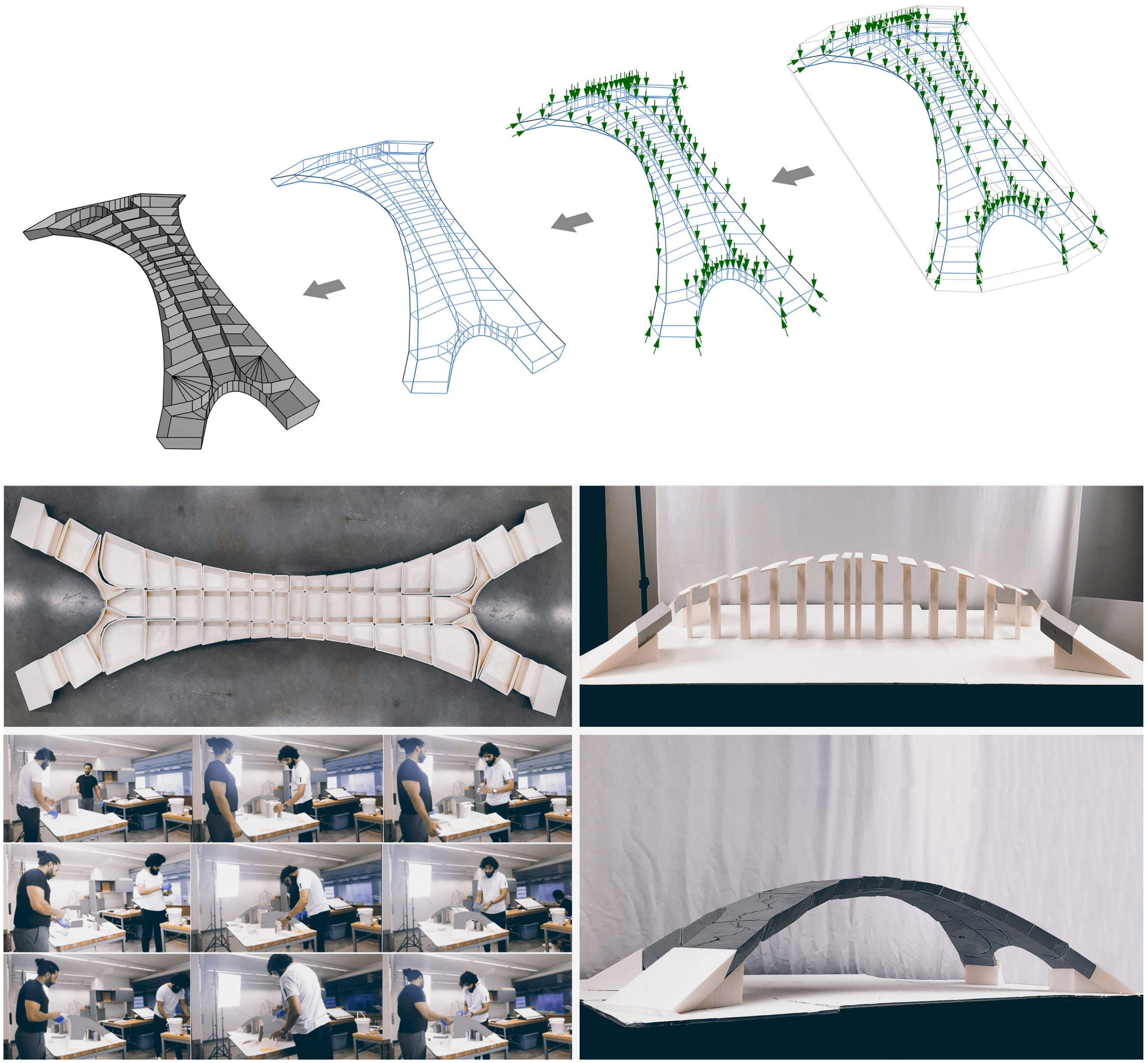


Figure 5. Creating the mold pieces from the Force Diagram, 3-D printed molds, formwork, assembly process, and assembled bridge.

assembly, all the supports were fixed on their positions and the formworks were also located under the bridge. From both sides units were placed on the supports and then the middle row of the bridge was constructed. Plaster was used as the mortar between the units. After finishing the middle row, the left and right wings were positioned in place and at the end the formwork was removed one by one starting from the center.

## 4. OTHER SKETCHES

In another work of Da Vinci there is a sketch of multiple triangles as a footnote [Figure 6]. A couple of interesting observation can be explained in this sketch. First, one of the edges has been divided into smaller segments that usually will be used to measure something. It is also apparent that the axes of the sketch have been unitized with showing the center point as O. This could be a classic representation of force diagram in 2D for a three members under two applied forces. Although this could be simply a geometrical representation of something, the aforementioned observations cannot be also ignored. this would help to understand the fact that he might have actually used it to calculate the internal forces and another sign that make us believe he in fact, was aware of the reciprocity between of force and form.

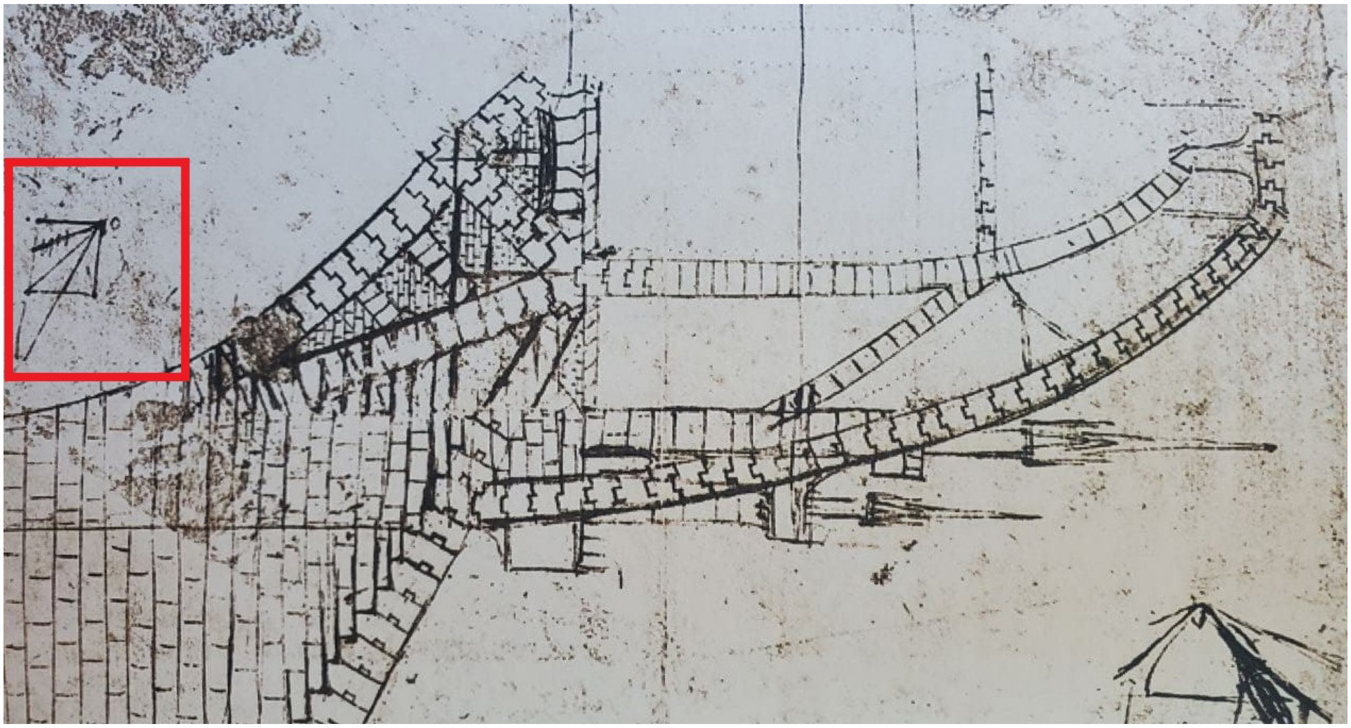


Figure 6. Illustration of a calibrated triangles in one of Da Vinci’s architectural design.

## 5. CONCLUSION AND FUTURE WORK

In this paper one of the Da Vinci’s controversial designs was investigate. The fact that his design was three-dimensional double curvature bridge intrigued us to investigate how he actually found the form of the bridge. Many historian believes he had no mathematical or geometrical calculation for this bridge. We have used our computational tools to check the accuracy of the form with his hand sketch. Surprisingly the result of our method was exactly the same as his design which raise the question that how he did that without using geometry. Our results show that, even if he did not directly use mathematics or show his geometrical findings, it seems he has done it in his mind and the design is intuitive and based on his mathematical cognitive ability. This research further continues to explore the potential of Da Vinci’s design with the use of modern materials and methods of construction to see how the design would have been built in our modern time. Furthermore, to pay homage to the genius mind of Da Vinci in realizing such an advanced design centuries ago, and to speculate on what he would have done using the means of our digital age.

## ACKNOWLEDGMENT

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